JOURNAL OF AIRCRAFT
Vol. 43, No. 3, May–June 2006

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Refining Satellite Methods for Pitot-Static Calibration

Robert J. Niewoehner*
United States Naval Academy, Annapolis, Maryland 21402

Nomenclature

$F_i(\bar{x})$	=	error from fit circle to ith data point
f, g	=	generic functions
$H(\bar{x})$	=	optimization cost function
$h_{p,c}$	=	calibrated pressure altitude
$h_{p,i}$	=	indicated pressure altitude
$J(\bar{x})$	=	Jacobian matrix
p_a	=	ambient pressure
p_s	=	(sensed) static pressure
V_c	=	calibrated airspeed
V_i	=	indicated airspeed
V_{gps}	=	measured groundspeed
V_T	=	estimated true airspeed
X_i	=	east–west component of measured ground speed, ith data point
X_w	=	east–west component of estimated wind speed
$\bar{x}$	=	estimate state vector
Y_i	=	north–south component of measured ground speed, ith data point
Y_w	=	north–south component of estimated wind speed
Δh_{pos}	=	altitude position error
Δp	=	static source position error
ΔV_{pos}	=	airspeed position error
ψ_{gps}	=	global positioning satellite ground-track azimuth angle
ψ_{wind}	=	wind direction

Introduction

THE advent of the global positioning satellite system (GPS), with its accuracy and inexpensive GPS receivers, quickly spurred innovative thinking in its application to the time–space–position instrumentation challenges required by flight test. The pitot-static calibration is among the most basic of flight-test practices because of the criticality of credible air data. Accurate calibration of an airplane's static source requires either true tapeline altitude, or true airspeed. Legacy methods have been either expensive or ungainly. Consequently, a variety of GPS methods have been developed and explored for use by general aviation practitioners. This Note describes the most reliable of current methods and an improved algorithm for the determination of true airspeed from GPS ground-track and ground-speed data.

Received 20 July 2005; revision received 25 August 2005; accepted for publication 25 August 2005.

*Director, Aeronautics, 590 Holloway Road, Code 11B. Member AIAA.

The airplane's influence on its own static ports necessitates pitot static calibration. Those systems rely on a total pressure sensor (typically a pitot tube) and flush-mounted static pressure ports to measure both indicated airspeed and indicated pressure altitude. The altimeter depends on the static pressure alone, whereas the indicated airspeed functionally measures the difference between the total and static pressures (the impact pressure). The total pressure can very accurately be sensed at subsonic speeds for reasonable ranges of angle of attack and sideslip, provided the pitot tube is well removed from the influence of boundary layers and upstream propulsion.[1,2] Pitot tube calibration is, therefore, commonly accepted as unnecessary (which is not the case for transonic or rotorcraft applications where the pitot tube sees nonisentropic flow). The same cannot be said for the static port, where the airplane introduces a pressure field that interferes with the ambient pressure measurement. The difference between the static pressure and ambient pressure is called the static source error or position error ($\Delta p_{\text{pos}} = p_s - p_a$). This static source error then appears in the cockpit instrumentation as both an altitude position error and an airspeed position error,

$$\Delta h_{\text{pos}} = h_{pc} - h_{pi} = f(\Delta p_{\text{pos}}) \tag{1}$$

$$\Delta V_{\text{pos}} = V_c - V_i = g(\Delta p_{\text{pos}}) \tag{2}$$

The manifestation of the static source error in both the altimeter and the airspeed indicator provides two avenues by which the error might be measured. Calibration methods for an airplane's pitot-static system, therefore, require either comparing true altitude with the sensed altitude, then finding Δp from Δh_{pos}, or measuring and comparing the sensed airspeed and true airspeed, then determining Δp from ΔV_{pos}. In Refs. 1 and 2, many of these methods are cataloged.

Each of the legacy methods has its attendant limitations. Well-resourced industry or government programs typically use multiple methods both to confirm results, as well cover the full flight envelope. Smaller airplane programs have been compelled to live with the limitations of the lower cost alternatives.

Handheld GPS units can now provide an extremely low-cost solution suitable for most applications, rivaling the accuracy of the most expensive methods. (High-altitude or supersonic applications may necessitate more elegant and sophisticated differential GPS/inertial navigation system methods beyond the scope of this discussion.) The GPS methods appropriate to general aviation applications are true airspeed methods, where the GPS ground speed is used to determine the true airspeed and, then, the calibrated airspeed. The difference between the calibrated and indicated airspeeds provides the velocity position error, from which the static source error and altitude position error can then be determined.

Discussion

GPS methods for determining true airspeed have grown in their complexity and accuracy. Original GPS solutions imitated the legacy ground-track method only using GPS in lieu of a surveyed ground course.[3–7] This step offered improvement because testing was no longer tied to a surveyed course and could instead be performed practically anywhere and at any altitude. With or without GPS, the method depends on flying reciprocal tracks over a geographical location, presuming that the reciprocal heading adequately neutralizes the effect of winds. Cross-track winds, however, can introduce

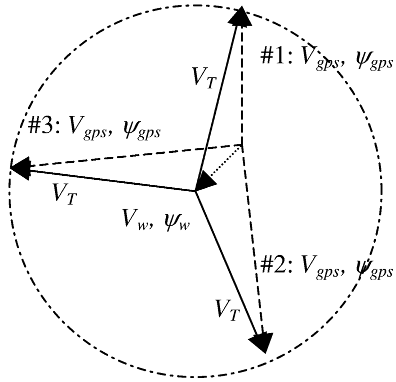

Fig. 1 Wind and V_T from GPS ground speed and ground track.

errors close to the magnitudes of the very phenomena we are trying to measure.[8] The horseshoe heading method was then devised, in which three tracks separated by 90–120 deg provide isolation of the wind magnitude and direction and, hence, resolve the true airspeed.[9–12] Several of these researchers encouraged a cloverleaf flight trajectory, intending to capture the data during passes through the same airmass. In Ref. 5, the method's legitimacy and accuracy was demonstrated against legacy methods.

This latter method records the GPS ground track and ground speed on three legs, ideally separated by 90–120 deg, at a constant airspeed and altitude. GPS units determine ground speed independently of the position and with much higher precision, using the satellites' Doppler shifts rather than time coding. Precisions to less than 0.1 kn are typically advertised, and some algorithms are capable of precisions better than 1 cm/s even in a severe multipath environment.[13] These accuracies are well within the desired tolerances for calibrating instruments that can typically be read to no better than the nearest knot. The units actually measure the speed of the GPS unit relative to the satellites and then calculate the ground speed from the known satellite velocity vectors and positions. Because the ground speeds of light aircraft are tiny compared to the measured relative velocities, the accuracy is not a function of the aircraft's ground speed. The airplane need only be stabilized for several seconds on each track, to allow any Kalman filtering to settle, and the method can be flown at any altitude. Data can be captured very quickly with the principle delay between points being the time required to establish at a new airspeed. For any given airspeed, those tracks and speeds can be represented by three vectors (Fig. 1, adapted from Ref. 10). Note that the wind direction in Fig. 1 is shown as the direction its going vs the meteorological convention of describing its originating direction. The numerical challenge is then determining the unique circle passing through the head of the three vectors. In Refs. 10–12 several approaches are presented. If we assume that the wind is invariant between the three legs, then the radius of that circle is the true airspeed and the center of the circle represents the wind speed and direction. In Ref. 10, it is commented that this method can be varied: Fly four successive legs at the same airspeed, calculate the results for the four possible combinations of three legs, and average the results. To distinguish this approach from the variation we propose, we call this the 3×4 method, meaning that four legs are flown resulting in four combinations of three legs. The National Test Pilot School distributes an Excel spreadsheet that determines the true airspeed by both approaches, as well as the complete pitot-static data reduction (Ref. 10).

In practice, four legs are indispensable to establishing data quality, though the proposed averaging scheme can be improved as will be outlined. Given the practical challenge of holding airspeed precisely for each of three legs, any three-leg method provides no direct quantitative measure of the pilot's success or the consistency of the air mass. Variations must be inferred from successive data points; if the winds vary wildly between data points, the data are suspect. Conversely, consistent winds from point to point suggest tight data. The problem lies in defining consistent winds and, if the winds vary from point to point, determining which points might still be trusted. If the winds vary no more than 2–3 kn between points, then that would suggest good consistency, but even those variations are larger than the desired accuracy of the calibration (nominally ± 1 kn).

The four-leg method provides a direct measure of the quality of each point in the scatter of the results obtained over the four combinations of three legs. In smooth conditions, we have routinely seen the four wind calculations cluster within 0.2 kn of one another, well within the 1-kn measurement precision of many GPS units and much tighter than the 1-kn tolerances within which the pilot was attempting to fly. In turbulent conditions, variations in the measured true airspeed on the order of 2–4 kn indicate that the point was poorly flown, or the air mass was not suitably uniform (the foundational assumption of all GPS methods). This quality check is not available if only three legs are flown. Flight-test examples that follow will illustrate the value.

The numerical processing is the one element of the four-leg method (the 3×4 algorithm) of Ref. 10 that warrants refinement. Assume four legs are flown at one airspeed, represented as legs A, B, C, and D, all roughly 90 deg apart. In Ref. 10, it is suggested that the true airspeed and winds be determined for the four possible triplets, (ABC), (ABD), (ACD), and (BCD), using the three-leg algorithm and then arithmetically averaged.

This approach has two weaknesses. First, the airspeed determined by averaging the four three-leg values will not be a genuine best fit to the data. Second, the winds are determined by averaging the four wind speeds and averaging the four wind directions. This is not a legitimate mathematical operation for vectors in polar coordinates, and we have seen results in wind values 90 or 180 deg from the true mean when the results for the four wind directions are distributed either side of north. Although this later flaw is easily amended, the algorithm's complexity then rivals the algorithm to follow.

Instead, a nonlinear least-squares method provides a more robust process, finding the one circle that best fits all four vectors simultaneously. (A MATLAB® function file executing this algorithm is available from the author.) Consider a scalar objective function $H(\bar{x})$, which is the sum of the squares of the error between an estimated circle and the four vector heads,

$$H(\bar{x}) = \sum_i F_i^2(\bar{x}) \tag{3}$$

where the state vector $\bar{x}$ describes the radius V_T, the center of the circle (X_w, Y_w),

$$\bar{x} = \begin{bmatrix} V_T \\ X_w \\ Y_w \end{bmatrix} \tag{4}$$

and the objective function $F_i(\bar{x})$ is the distance from the ith vector, expressed in Cartesian coordinates (X_i, Y_i), to the circle

$$F_i(\bar{x}) = V_T - \sqrt{(X_i - X_w)^2 + (Y_i - Y_w)^2} \tag{5}$$

An iterative Newton-method search can then find the least-squares solution in several iterations,

$$\bar{x}_{k+1} = \bar{x}_k - [J(\bar{x})^T J(\bar{x})]^{-1} J(\bar{x})^T F(\bar{x}) \tag{6}$$

Note that in most computational software, for example, MATLAB, the pseudoinverse in Eq. 6 is more efficiently calculated with a operator: $\bar{x}_{k+1} = \bar{x}_k - J(\bar{x}) \backslash F(\bar{x})$. The Jacobian $J(\bar{x})$ and its components are defined as

$$J(\bar{x}) = \begin{bmatrix} \dfrac{\partial F_1}{\partial V_T} & \dfrac{\partial F_1}{\partial X_w} & \dfrac{\partial F_1}{\partial Y_w} \\ \dfrac{\partial F_2}{\partial V_T} & \dfrac{\partial F_2}{\partial X_w} & \dfrac{\partial F_2}{\partial Y_w} \\ \dfrac{\partial F_3}{\partial V_T} & \dfrac{\partial F_3}{\partial X_w} & \dfrac{\partial F_3}{\partial Y_w} \\ \dfrac{\partial F_4}{\partial V_T} & \dfrac{\partial F_4}{\partial X_w} & \dfrac{\partial F_4}{\partial Y_w} \end{bmatrix} \tag{7}$$

$$\frac{\partial F_i}{\partial V_T} = 1, \qquad \frac{\partial F_i}{\partial X_w} = \frac{(X_i - X_w)}{\sqrt{(X_i - X_w)^2 + (Y_i - Y_w)^2}}$$

$$\frac{\partial F_i}{\partial Y_w} = \frac{(Y_i - Y_w)}{\sqrt{(X_i - X_w)^2 + (Y_i - Y_w)^2}} \tag{8}$$

If the state vector is initialized with the radius set to the average of the four ground speeds and the circle's center at the geometric center of the four vectors, then two iterations are typically required to converge to less than 0.01 kn. The data quality is expressed by the root mean square error,

$$\text{rms} = \sqrt{H(\bar{x})/4} \tag{9}$$

Experimental Results

The results to follow are for a general aviation airplane in two different configurations and collected on two separate days with varying atmospheric conditions. The GPS ground speed was determined with a GARMIN 92 handheld, which has both an advertised and demonstrated accuracy of less than 0.1 kn below 100 kn. Above 100 kn, the unit rounded to the nearest 1 kn. Data set 1 was gathered in smooth air, whereas data set 2 was collected on a turbulent day, where the pilot struggled to maintain the desired airspeed tolerance (±1 kn). Table 1 lists the data for several airspeeds collected on the same sortie, with the resultant winds and true airspeeds calculated by both the 3 × 4 method and the preceding nonlinear least-squares algorithm. The 3 × 4 method returns a standard deviation of the four airspeeds produced by the four triplets, whereas the least-squares algorithm returns the rms error. For the Table 1 data, both these data quality measures return almost identical values. These points demonstrate an excellent fit using both algorithms, though the rms error is consistently smaller than the error represented by the standard deviation found by averaging over the four legs reduced as triplets. Consistent winds are seen point to point, providing very high data confidence and indicating that the winds were stable during the test sortie. Furthermore, the tight tolerances observed had been obtained by flying a pure box pattern with cardinal headings, with no attempt to clover leaf back through the identical air mass, indicating that the clover-leaf maneuver is unnecessarily expensive. Consequently, a superior, higher confidence data product can be obtained with less test time by omitting the clover-leaf three-leg pattern and adding a fourth data leg.

In contrast, Table 2 lists the data taken under gusty conditions. The four points presented are selected to illustrate the full spectrum of that day's results. Data quality varies significantly from point to point. Point 3 exhibits high data quality, despite the conditions, with an rms error for the fit of well less than 1 kn. Points 1 and 2, however, are much more ragged, with rms errors of 2.16 and 1.97 kn. Depending on the application, this may or may not represent adequate accuracy, but the four-leg methods permit the test team to

Table 1 Data set 1 (smooth conditions)

	Point 1		Point 2		Point 3	
Result	V_{gps}, kn	ψ	V_{gps}, kn	ψ	V_{gps}, kn	ψ
			GPS data			
Leg 1	66.7	338	105	252	100	342
Leg 2	72.1	90	107	168	103	88
Leg 3	98.1	175	86.3	87	129	176
Leg 4	93.0	250	80.2	338	128	250
			3 × 4 Method			
V_T	81.4		94.1		113.9	
σ	0.11		0.91		0.28	
Wind	19.2	22.3	17.2	22.0	19.0	29.7
			Least squares method			
V_T	81.4		94.1		113.9	
RMS	0.098		0.79		0.24	
Wind	19.2	22.3	17.2	22.2	19	29.7

Table 2 Data set 2 (gusty conditions)

	Point 1		Point 2		Point 3		Point 4	
Result	V_{gps}, kn	ψ	V_{gps}, kn	ψ	V_{gps}, kn	ψ	V_{gps}, kn	ψ
				GPS data				
Leg 1	99	82	101	337	116	87	80	340
Leg 2	123	170	112	85	141	174	92	87
Leg 3	111	250	131	174	128	256	118	172
Leg 4	95	343	127	254	105	346	105	245
				3 × 4 Method				
V_T	106.7		117.1		122.2		97.6	
σ	2.51		2.31		0.65		1.08	
Wind	15.4	9.0	17.6	15.4	19.1	9.6	20.1	4.0
				Least squares method				
V_T	106.6		117		122.2		97.6	
RMS	2.16		1.97		0.56		0.92	
Wind	15.2	9.1	17.1	14.7	19.2	9.3	20	4.7

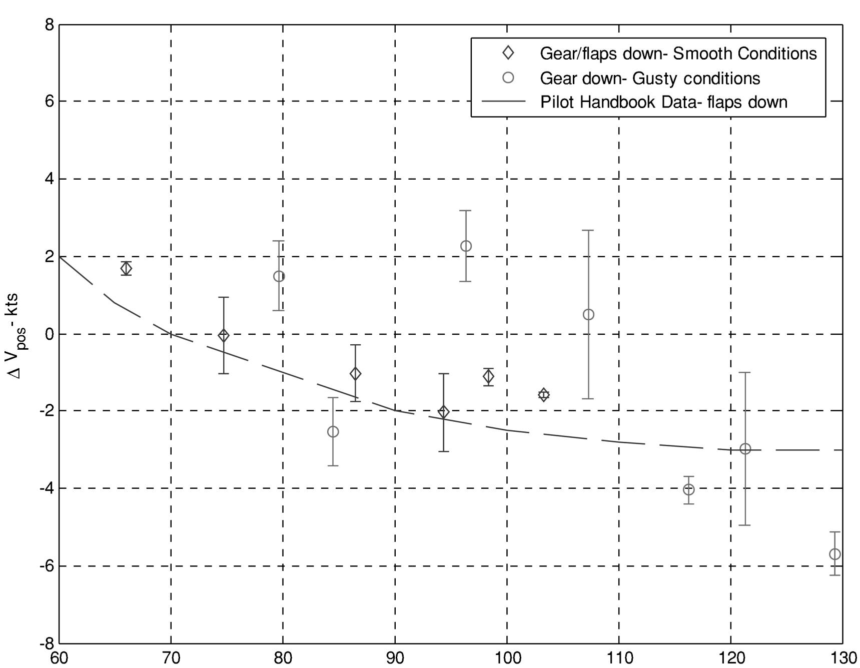

Fig. 2 A-36 Bonanza velocity position error.

quantify the accuracy of their result. The value of the nonlinear least-squares reduction approach is seen by tighter fits across the points. The fourth data point illustrates the value. Had the test team's threshold been accuracies of 1 kn or better, the nonlinear fit would score this point as complete, whereas the 3×4 reduction method would require repeating the point (unnecessarily).

Figure 2 shows the A-36 Bonanza's resultant velocity position errors for the two complete data sets, as determined by the least-squares search using the four legs. The production airspeed indicator was calibrated on the ground and the calibration applied. The error bars indicate the rms error after the fit. An additional $\frac{1}{2}$ kn of uncorrelated error arises from the resolution of the airspeed indicator and an additional $\frac{1}{2}$ kn of correlated error for ground speeds over 100 due to the rounding in the Garmin 92. Both of these latter two errors are not reflections on the algorithm, rather the specific equipment and can be tightened with different equipment selections. The value of the four-leg method is evident due to its provision of an internal check of the data quality that does not exist with three-leg methods. These error bars then flag the reliability of the first data set, while casting doubt on the validity of the second. As a further check of the algorithm's consistency, the manufacturer pilot handbook data is plotted.[14]

Conclusions

With the methods described, GPS provides a means to capture quickly and inexpensively the true airspeed data necessary for the calibration of pitot-static systems. With the result of improving on previously published techniques, the inclusion of a fourth data leg, and the employment of a simple nonlinear least-squares algorithm, permits determination of the both the true airspeed and an rms measure of the data quality. This results in both improved accuracy and quantified confidence in the result. Furthermore, representative data indicate that the time-consuming process of adding clover-leaf turns is unnecessarily costly.

Acknowledgments

The author is particularly indebted to G. V. Lewis and the National Test Pilot School for openly distributing his software for use by both students and flight-test practitioners.

References

[1] Ward, D., and Straganic, T., *Flight Test Engineering*, 2nd ed., Kendall–Hunt, Dubuque, IA, 1996, Chap 2.

[2] McCue, J., *Pitot–Static Systems*, 2nd ed., U.S. Naval Test Pilot School, Patuxent River, MD, 1994, p. 5–5.

[3] Fox, D., "Is Your Speed True?," *Kitplanes*, Fisher Flying Products, Edgeley, ND, Feb. 1995, pp. 49, 50.

[4] Smith, H. C., "Procedures for Airspeed Calibration by use of GPS," *Proceedings AIAA/FAA Joint Symposium on General Aviation Systems*, May 1994; also Dept. of Transportation/Federal Aviation Administration/CT, Tech Rept. DOT/FAA/CT-94/63, Washington, DC, 1994, pp. 597–602.

[5] Lewis, G.V., "A Flight Test Technique Using GPS for Position Error Correction Testing," *Cockpit, Quarterly of the Society of Experimental Test Pilots*, Jan/Feb/Mar, 1997, pp. 20–24.

[6] Bailey, W. D., "Investigation of Using Global Positioning System for Air Data System Calibration of General Aviation Aircraft (Have Pacer II)," Rept. AFFTC-TR-95-76, Air Force Flight Test Center, Edwards, CA, Jan. 1996.

[7] "Flight Test Guide for Certification of Part 23 Airplanes," Federal Aviation Administration, Advisory Circular 23-8B, Washington, DC, Aug. 2003, pp. 139, 140, A9-12ff.

[8] Rogers, D. F., "ASI Calibration," *Newsletter of the World Beechcraft Society*, Vol. 12, No. 4, 2000, pp. 10, 11, 37.

[9] Gray, D., "Using GPS to Accurately Establish True Airspeed (TAS)," National Test Pilot School, June 1998, URL: www.ntps.edu/Files/tas_fnl3.pdf [cited 25 Aug. 2005].

[10] Lewis, G. V., "Using GPS to Determine Pitot Static Errors," National Test Pilot School, Aug. 2003, URL: www.ntps.edu/Files/GPS%PEC%20Method.doc [cited 25 Aug. 2005].

[11] Olson, W., "Pitot–Static Calibration Using a GPS Multi-Track Method," *Proceedings of the 29th Symposium of the Society of Flight Test Engineers*, Paper No. 29-17, Society of Flight Test Engineers, Lancaster, CA, 1998.

[12] Rogers, D. F., "An Engineering Flight-Test Course Emphasizing Flight Mechanics Concepts," *Journal of Aircraft*, Vol. 39, No. 1, 2002, pp. 82, 83.

[13] Serrano, L., Kim, D., Langley, R. B., Itani, K., and Ueno, M., "A GPS Velocity Sensor: How Accurate Can It Be?—A First Look," *Proceedings of the National Technical Meeting*, Inst. of Navigation, Fairfax, VA, Jan. 2004, p. 875.

[14] *Bonanza A-36 Pilot's Operating Handbook*, Raytheon Corp., Wichita, KS, July 1994, pp. 5–10.